HEWLETT-PACKARD
JOURNAL
TECHNICAL INFORMATION FROM THE -hp- LABORATORIES
Vol. 11 No. 8-10
PUBLISHED BY THE HEWLETT-PACKARD COMPANY, 1501 PAGE MILL ROAD, PALO ALTO, CALIFORNIA
APR.-JUNE, 1960

The Effect of μ-Circuit Non-Linearity on the Amplitude Stability of RC Oscillators

Ed. Note: *In the RC oscillator the amplitude of oscillation is controlled by a thermally-sensitive resistance in the form of a small tungsten lamp. This control element gives the oscillator its characteristic constancy of output level over a wide range of operating conditions. In practice, for example, the oscillator's response is quite slight to such disturbances as hum, microphonics, a change with tuning in the impedance ratios in the frequency-determining network, or a change in loading at the oscillator output.*

At the same time, and in contrast to the great majority of cases, experience with the RC oscillator has disclosed instances in which the oscillator has exhibited an exaggerated response to such disturbances (see Fig. 2 below). In a few cases actual envelope instability has been observed. Although this effect was generally attributed to tube anomalies, since aging or replacing the tubes removed the effect, over a period of time the impression began to form that a more definite factor was involved.

The envelope response was then investigated analytically by Bernard M. Oliver, -hp- vice-president for R & D, and his analysis of the RC oscillator envelope response to amplitude disturbances introduced into the control element is presented in the following article. We believe the analysis will be of general interest because it takes into account the effect of the small non-linearity that exists in the amplifier portion of the oscillator circuit, where previous analyses have assumed an ideally linear amplifier. The analysis shows that, if the amplifier were in fact ideally linear, the envelope response to amplitude disturbances would be some 100 db more than it is. It shows further that the amplifier must have a slight compression for the oscillator to be practical. This agrees with observed results.

In the years since the introduction of the RC oscillator, many analyses have been made of the power-sensitive feedback-controlling mechanism that regulates the level of oscillation. To the author's knowledge, these analyses have all assumed perfect linearity in the amplifier circuit. Predicted performance, however, does not always agree with experience. In particular, the envelope stability is usually much better (i.e., transient disturbances in oscillation amplitude die out much more rapidly) than the linear theory predicts, especially for the higher oscillator frequency ranges. The analysis presented here includes the effect of non-linearity in the amplifier, and yields results which are in close agreement with observed performance.

SEE ALSO:
Using VLF Standard Broadcasts .... p. 8
De Gaulle visits -hp- p. 3

It is shown that amplifier non-linearity so slight as to cause insignificant waveform distortion has a profound effect on envelope stability. In fact, were it not for this slight non-linearity, it would be virtually impossible to build a simple lamp-stabilized RC oscillator with good envelope stability over a wide frequency range. This is borne out by the fact that occasionally oscillators show up in production which, because of a fortuitous combination of tube characteristics, exhibit extremely low distortion. Invariably these units give trouble with envelope "bounce", i.e., they have a highly oscillatory response to envelope disturbances. In some cases, actual instability occurs

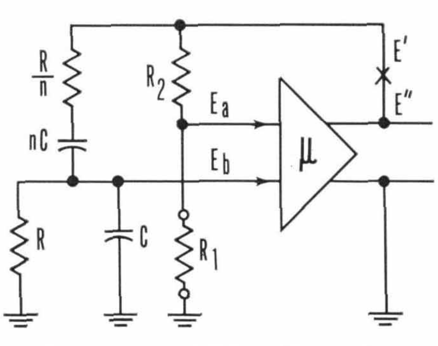


Fig. 1. *Basic circuit of RC oscillator. Constancy of output level is achieved by controlling amplitude of oscillation with thermally-sensitive resistance R_1, a small tungsten lamp in negative feedback circuit.*

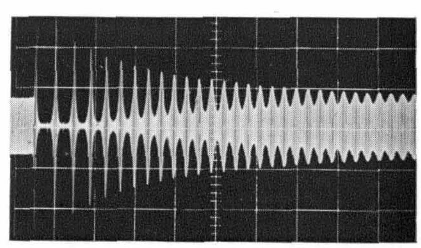


Fig. 2. *Oscillogram showing unusual squegging-type transient response produced in RC oscillator by making linearity of amplifier portion of circuit extremely high. Slight amplifier compression gives normal response shown in Fig. 7(a).*

PRINTED IN U.S.A.

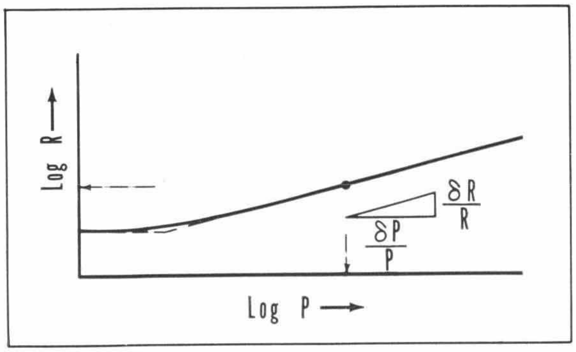


Fig. 3. *Plot of resistance of tungsten lamp as a function of applied power.*

and a modulated output is produced.

In the following analysis we shall begin by determining the normal conditions for steady-state oscillation and the resulting lamp resistance. We next determine the perturbation in lamp resistance produced by a perturbation in power fed to the lamp circuit. Finally, we determine the perturbation in oscillator output power resulting from a given perturbation in the lamp resistance. The product of these last two transmissions is the envelope-loop gain and obeys the usual Nyquist criterion for stability.

BASIC OSCILLATOR EQUATIONS

Figure 1 shows a typical RC oscillator. The amplifier is assumed to be differential so that the output

$$E = \mu\,(E_b - E_a).$$

We shall assume that μ is positive, real, and independent of frequency. R_1 is the lamp, or other thermally sensitive element with a positive temperature coefficient. (The same analysis will hold if R_2 has a negative temperature coefficient.) Let us assume the loop to be broken at the point "X". Then

$$\frac{E_b}{E'} = \frac{\dfrac{R}{pCR+1}}{\dfrac{R}{pCR+1} + \dfrac{1}{n}\left(R + \dfrac{1}{pC}\right)} = \frac{nz}{z^2 + (n+2)z + 1} \qquad (1)$$

where

$$z = pCR = \frac{p}{\omega_0} = j\frac{\omega}{\omega_0}, \quad \omega_0 \equiv \frac{1}{RC} \qquad (2)$$

Let

$$k = \frac{E_a}{E'} = \frac{R_1}{R_1 + R_2} \qquad (3)$$

Then

$$\beta \equiv \frac{E_b - E_a}{E'} = \frac{nz}{z^2 + (n+2)z + 1} - k \qquad (4)$$

The normal modes of the system are given by the roots of $1 - \mu\beta = 0$, and are

$$z_{1,2} = -\frac{b}{2} \pm j\sqrt{1 - \left(\frac{b}{2}\right)^2} = -\frac{b}{2} \pm j\lambda \qquad (5)$$

where

$$b = (n+2) - \frac{n}{k + \dfrac{1}{\mu}} \qquad (6)$$

The corresponding time function has the form

$$e = A\exp\left(-\frac{b}{2}\omega_0 t\right)\cos(\lambda\omega_0 t + \phi) \qquad (7)$$

For constant amplitude oscillations, we require $b = 0$ and therefore from (6):

$$k = \frac{n}{n+2} - \frac{1}{\mu} \equiv k_0 \qquad (8)$$

Substituting this result in (3) we find the operating resistance, $\overline{R}_1$, of the lamp to be

$$\frac{\overline{R}_1}{R_2} = \frac{k_0}{1 - k_0} = \frac{n\mu - (n+2)}{2\mu + (n+2)} \qquad (9)$$

LAMP DYNAMICS

If a plot on log paper is made of the resistance vs. power input for a vacuum tungsten lamp, the result will consist of two fairly straight line portions: a constant resistance portion (at very low power inputs) and a sloping portion as shown in Fig. 3. From such a plot one may determine the slope

$$S_0 = \frac{d(\log R)}{d(\log P)} = \frac{\dfrac{\delta R}{R}}{\dfrac{\delta P}{P}} \qquad (10)$$

S_0 describes the resistance sensitivity of the lamp and is a factor in the envelope loop gain. Typically

$$S_0 \approx \frac{1}{4}$$

The resistance change of the lamp is, of course, due to temperature change of the filament. The power, P_ℓ, lost by radiation and conduction from the filament is a non-linear function of temperature

$$P_\ell = f(T)$$

However, at any operating temperature $f(T)$ will have a well-defined slope:

$$G_T = \frac{dP_\ell}{dT} = f'(T)\Big|_{T = T_0}$$

G_T may be thought of as a thermal conductance.

As the temperature is changing, a part, P_s, of the input power goes into heating the filament. Thus

$$P_s = C_T\frac{dT}{dt}$$

where C_T = thermal capacity of the filament
= (specific heat) × (mass).

The total power input divides between heat loss and heat being stored, so we have

$$\delta P = C_T\,\delta\dot{T} + G_T\,\delta T,$$

and if δP and δT are the amplitudes of sinusoidal perturbations, we may replace differentiation with multiplication by $p = j\omega$ to get:

$$\delta P = (pC_T + G_T)\,\delta T$$

$$\frac{\delta T(p)}{\delta P(p)} = \frac{1}{pC_T + G_T} \qquad (11)$$

High-frequency temperature variations are suppressed by a simple pole, the cutoff frequency being

$$\omega_T = \frac{G_T}{C_T}$$

Taking this fact into account, we see that the "transmission" of the lamp as a control element, i.e., its resistance response to input power fluctuations is given by

$$\frac{\dfrac{\delta R}{R}}{\dfrac{\delta P}{P}} = S_0\frac{\omega_T}{p + \omega_T} \qquad (12)$$

where δP is now the amplitude of a sinusoidal power variation at frequency ω, and $p = j\omega$.

Unless the lamp is driven from a matched source, the situation is complicated by the fact that the lamp power input is affected not only by changes in the source but also by the resulting change in the lamp resistance itself. If a lamp of resistance R_1 is driven by a Thévenin generator of peak voltage E and resistance R_2 (as in Fig. 1), the power input to the lamp is

$$P = \frac{I^2 R_1}{2} = \frac{R_1}{(R_1 + R_2)^2}\frac{E^2}{2} \qquad (13)$$

Thus

$$\delta P = \frac{\partial P}{\partial E^2}\,\delta(E^2) + \frac{\partial P}{\partial R_1}\,\delta R_1$$

$$= \frac{1}{2}\frac{R_1}{(R_1 + R_2)^2}\,\delta(E^2) - \frac{R_1 - R_2}{(R_1 + R_2)^3}\frac{E^2}{2}\,\delta R_1$$

Dividing this by (13) we find

$$\frac{\delta P}{P} = \frac{\delta(E^2)}{E^2} - \rho\frac{\delta R_1}{R_1} \qquad (14)$$

where

$$\rho = \frac{R_1 - R_2}{R_1 + R_2} = (2k - 1)$$

Combining (14) and (12) to eliminate $\delta P/P$ we obtain

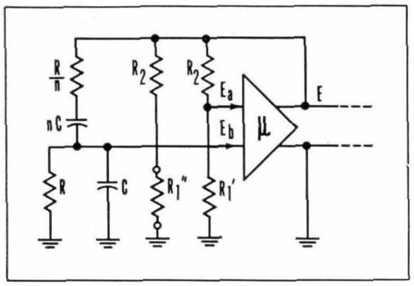


Fig. 4. *Circuit model used for investigating envelope loop gain.*

$$\frac{\delta R_1}{R_1} = S_0 \frac{\omega_T}{p + (1 + S_0\rho)\,\omega_T}\,\frac{d(E^2)}{E^2} \qquad (15)$$

which still exhibits a simple pole, but with altered cutoff frequency and low frequency transmission. Letting

$$S = \frac{S_0}{1 + S_0\rho}$$

$$\omega_\ell = (1 + S_0\rho)\,\omega_T$$

then the transmission, $G(p)$, of the lamp circuit to amplitude changes is simply

$$G(p) \equiv \frac{\dfrac{\delta R_1}{R_1}}{\dfrac{\delta(E^2)}{E^2}} = S\,\frac{\omega_\ell}{p + \omega_\ell} \qquad (16)$$

We note that if $\rho = 0$, corresponding to a matched source, $S = S_0$ and $\omega_\ell = \omega_T$. This is the case if μ is high and $n = 2$ (see equation (8)). With a "voltage" source ($R_2 < R_1$, $\rho > 0$) the increase in lamp resistance decreases the power input, resulting in less total change of resistance and a faster convergence to the final value. With a "current" source ($R_2 > R_1$, $\rho < 0$) an increase in lamp resistance causes a further increase in power input, so the total resistance change and time to get there are increased. Regardless of the source impedance the high frequency transmission is $S_0\omega_T/p$ (see eq. (15)).

We must distinguish between the power fluctuations which occur during each cycle of the oscillation and the (slower) changes resulting from fluctuations in the envelope itself. The heat dissipation of the filament $f(T)$ is of the form

$$f(T) = A(T^4 - T_a^4) + B(T - T_a)$$

where A and B are constants and T_a is the ambient temperature. The first term represents radiation, and the second term conduction loss. Thus

$$G_T = 4AT^3 + B$$

and increases rapidly with temperature. In a properly designed RC oscillator the lamp is operated at a temperature high enough to be on the sloping portion of Fig. 3 and low enough so that ω_T is well below the bottom of the oscillator tuning range. Thus the thermal time constant prevents appreciable resistance changes during the oscillation cycle (the power fluctations are at double frequency if there is no d-c in the lamp) and harmonic distortion from this cause is reduced. As a consequence we need be concerned only with resistance changes arising from changes in envelope amplitude.

ENVELOPE LOOP GAIN—LINEAR μ-CIRCUIT

In determining the envelope loop gain it is convenient to break only the envelope feedback loop. This can be done, in effect, by duplicating the resistance arms of the Wien bridge as shown in Fig. 4. The negative feedback is taken from a branch containing a resistor R_2 and a non-thermally sensitive resistor R_1', while the branch containing R_2 and the lamp is left connected across the output as before. We neglect the additional loading of the oscillator output by the additional branch since it is in fact not present under normal conditions. (If you like, you may consider the added branch to be driven through an ideal infinite input impedance unity gain amplifier.)

Initially the system is assumed to be oscillating stably with the output

$$e = E_1 \cos(\omega_0 t + \phi),$$

R_1' having been set at precisely $\overline{R_1}$, the value given by (9). Under these conditions R_1'' will also have the value $\overline{R_1}$. A perturbation $\delta R_1'$ is now introduced in R_1' and the resulting change of lamp resistance $\delta R_1''$ determined. The ratio $\delta R_1''/\delta R_1'$ is the envelope loop gain, H:

$$H = \frac{\delta R_1''}{\delta R_1'} = \frac{\dfrac{\delta R_1''}{\overline{R_1}}}{\dfrac{\delta R_1'}{\overline{R_1}}} = \left(\frac{\dfrac{\delta R_1''}{\overline{R_1}}}{\dfrac{\delta(E^2)}{E_1^2}}\right)\left(\frac{\dfrac{\delta(E^2)}{E_1^2}}{\dfrac{\delta R_1'}{\overline{R_1}}}\right) \qquad (17)$$

The first factor on the right is the perturbation in the lamp resistance pro-

DE GAULLE VISITS -*hp*- PLANT

Wm. R. Hewlett (right) shows France's President Charles De Gaulle (center) planned floor arrangement for new -hp- factory building under construction.

The Hewlett-Packard plant was a special point of interest for France's President Charles De Gaulle on his recent tour of the U. S. The -*hp*- plant, together with its location, the well-known Stanford Industrial Park, were visited by President De Gaulle as examples of leading present-day concepts relating the design of industrial plants and industrial areas.

President De Gaulle and his party were conducted on the tour of the -*hp*- plant and laboratories by -*hp*- Executive Vice-President Wm. R. Hewlett.

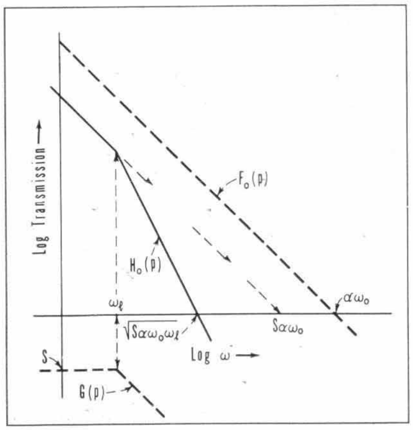


Fig. 5. Envelope loop gain characteristic $H_0(p)$ that would exist if amplifier were ideally linear. This characteristic exhibits considerable stretch of 12 db/octave gain slope above ω_ℓ, the frequency corresponding to the lamp circuit time constant, resulting in poor phase margin and oscillatory transient response.

duced by the perturbation in power input to the lamp branch and is given by the results of the previous section. The second factor on the right, the perturbation in amplifier power output produced by changes in the resistance R_1', we must now determine.

When R_1' is changed from $\overline{R_1}$ to $\overline{R_1} + \delta R_1'$, b changes from zero (representing steady state oscillation) to

$$b = \left[\left(\frac{\partial b}{\partial k}\right)\left(\overline{R_1}\,\frac{\partial k}{\partial R_1}\right)\right]_{\substack{R_1 = \overline{R_1} \\ k = k_0}} \frac{\delta R_1}{\overline{R_1}}$$

$$\blacksquare\ \alpha \frac{\delta R_1'}{\overline{R_1}}$$

By taking the derivatives of equations (6) and (3) and substituting in them relations (8) and (9) respectively, we find

$$\left.\begin{aligned} \alpha &= 2 + \frac{n^2-4}{n}\frac{1}{\mu} - \frac{(n+2)^2}{n}\frac{1}{\mu^2} \\ &\approx 2, \text{ for } \mu >> 1 \end{aligned}\right\} \quad (18)$$

Thus for a small constant increment,

$$\frac{\delta R_1'}{\overline{R_1}} = \epsilon\,,$$

the output will be

$$e = E_1 \exp\left(-\frac{\alpha\epsilon}{2}\,\omega_0 t\right)\cos(\lambda\omega_0 t + \phi)$$

The envelope therefore has the form

$$E = E_1 \exp\left(-\frac{\alpha\epsilon}{2}\,\omega_0 t\right),$$

and its square is

$$E^2 = E_1^2 \exp(-\alpha\epsilon\omega_0 t)$$

The change in E^2 is simply

$$\Delta(E^2) = E^2 - E_1^2$$

and so the response to a step of amplitude ϵ is:

$$\frac{\Delta(E^2)}{E_1^2} = \left[\exp(-\alpha\epsilon\omega_0 t) - 1\right] \quad (19)$$

This is the step response of a system whose transmission is given by

$$F_0(p) = -\frac{\alpha\omega_0}{p + \alpha\epsilon\omega_0}$$

corresponding to a real root at

$$p = -\alpha\epsilon\omega_0.$$

Note that the magnitude and sign of the root depend on ϵ. Hence for sinusoidal perturbations of R_1', the root oscillates about $p = 0$, and to an exceedingly good approximation, we may write

$$F_0(p) = -\frac{\alpha\omega_0}{p}\,. \quad (20)$$

Combining this result with (16) we find for the total envelope loop gain

$$H_0(p) = F_0(p)\,G(p) = -\frac{S\alpha\omega_0\omega_\ell}{p(p+\omega_\ell)}\,. \quad (21)$$

A typical loop gain characteristic is shown in Fig. 5. The loop gain is infinite at zero frequency and at low frequencies falls at 6 db/octave on a line which would intersect unity gain at $\omega = S\alpha\omega_0$. At ω_ℓ the additional pole due to the lamp circuit is felt, and above this frequency the gain falls at 12 db/octave.

Since for high oscillation frequencies $S\alpha\omega_0 >> \omega_\ell$, the linear analysis shows that there would be a long stretch of 12 db/octave gain slope before gain crossover and continuing indefinitely thereafter. This would result in extremely poor phase margin and a highly oscillatory transient response. We can, in fact, readily compute this response.

The transmission function relating disturbances in R_1, say, to disturbances in the output amplitude (squared) with the envelope loop closed is

$$T_0(p) = \frac{F_0(p)}{1 - H_0(p)} = -\frac{\dfrac{\alpha\omega_0}{p}}{1 + \dfrac{S\alpha\omega_0\omega_\ell}{p(p+\omega_\ell)}} \quad (22)$$

$$= -\frac{\alpha\omega_0(p+\omega_\ell)}{p^2 + \omega_\ell p + S\alpha\omega_0\omega_\ell}$$

The roots of the denominator are

$$P_{1,2} = -\frac{\omega_\ell}{2} \pm j\sqrt{S\alpha\omega_0\omega_\ell - \left(\frac{\omega_\ell}{2}\right)^2}$$

so that in response to a step or impulse disturbance the output amplitude (squared) will describe a damped oscillation of the form

$$E^2 = E_1^2 + A\exp\left(-\frac{\omega_\ell t}{2}\right)$$

$$\cdot \cos\left[\sqrt{S\alpha\omega_0\omega_\ell - \left(\frac{\omega_\ell}{2}\right)^2}\ t + \phi\right]$$

The "bounce" has a frequency

$$\sqrt{S\alpha\omega_0\omega_\ell - \left(\frac{\omega_\ell}{2}\right)^2}$$

and dies out with a time constant $2/\omega_\ell$, or twice the time constant of the lamp circuit. For $S\alpha\omega_0 >> \omega_\ell$, the Q of this oscillation is rather high, indicating considerable enhancement of disturbances at frequencies in the vicinity of

$$\omega\ \blacksquare\ \sqrt{S\alpha\omega_0\omega_\ell}$$

This can be seen directly from (22). Substitution of $p = \pm j\sqrt{S\alpha\omega_0\omega_\ell}$ yields

$$\left|T_0\right|_{max} \approx \frac{\alpha\omega_0}{\omega_\ell} \quad (23)$$

Thus in a typical case, if $S = 1/4$, $f_\ell = 0.2$ cps, $\alpha = 2$, and $f_0 = 500$ kc, we would have

$$|T_0| \approx \frac{2 \times 500{,}000}{.2} = 5 \times 10^6 \text{ or } 134 \text{ db!}$$

This would occur at a modulation frequency of 224 cps. At lower oscillator frequencies the enhancement is reduced somewhat but still is serious. For example, at 144 kc the enhancement is 123 db at the likely modulation frequency of 120 cps. It would seem from the linear theory that over the upper oscillator ranges the amplitude stabilization would be extremely troublesome with large amplification of circuit hum and microphonics.

While RC oscillators can be made which exhibit such behavior, it is by no means typical. The observed envelope transient response is in general non-oscillatory or only slightly so. At first one might suspect that the lamp transmission might be more complicated than that of a simple pole—that, in particular, it might fall off more slowly than 6 db/octave at high frequencies. Any slight decrease in the slope of $G(p)$ at high frequencies would increase the loop phase margin and greatly improve the stability. However, measurements on actual lamps indicate that any departures from the 6 db/octave slope, if present, are very slight. As we shall see in the next section, the discrepancies between the above linear theory and observation can be accounted for by the very slight non-linearity which in fact is present in the amplifier we have so far described by the constant, μ.

ENVELOPE LOOP GAIN—SLIGHTLY NON-LINEAR μ-CIRCUIT

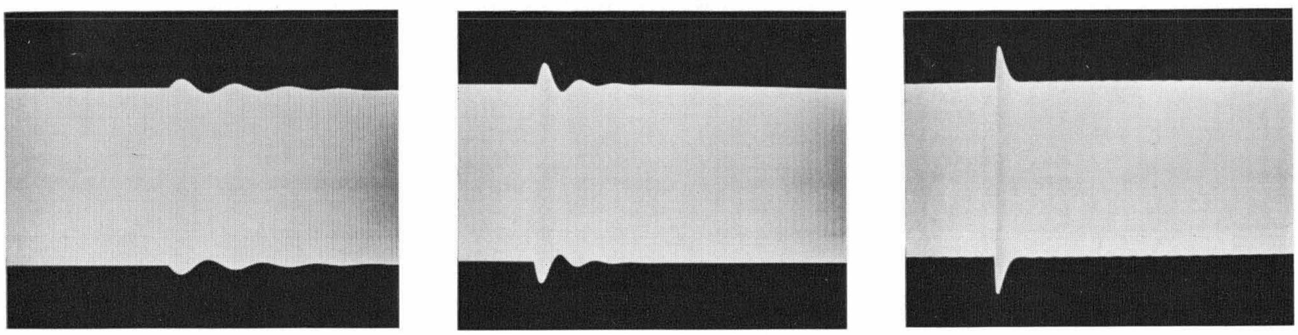
If μ decreases with an increase in oscillation level, then decreasing the resistance R_1' in Fig. 4 to a value slightly less than $\overline{R_1}$ would not cause the oscillation amplitude to increase indefinitely. Instead the oscillation would merely build up till μ decreased enough to make the new value of R_1' correct for stable oscillations (See eq. (8)). In principle a lamp would no longer be necessary, though the amplitude would be very sensitive to changes in μ, R_1, and R_2, as well as to tracking of the bridge capacitors and resistors. This same stabilizing action of the amplifier non-linearity is effective when a lamp is present and the loop is closed.

Normally, a non-linear transfer characteristic is described by expressing the instantaneous output as a series containing powers of the instantaneous input. Here we shall do the reverse, and represent the input as a series of powers of the output:

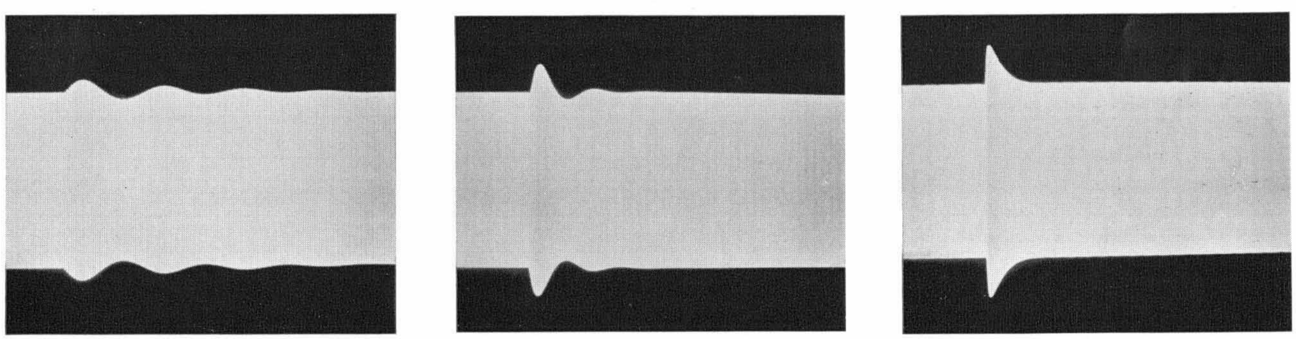
$$e_{in} = C_1 e_{out} + C_2 e^2_{out} + C_3 e^3_{out} + - - -$$

There are two reasons for doing this. First, in a feedback amplifier (and at harmonic frequencies an RC oscillator has large negative feedback) the distortion actually appears principally in the net input to the μ circuit. Second, the equations are simpler with this equally valid representation.

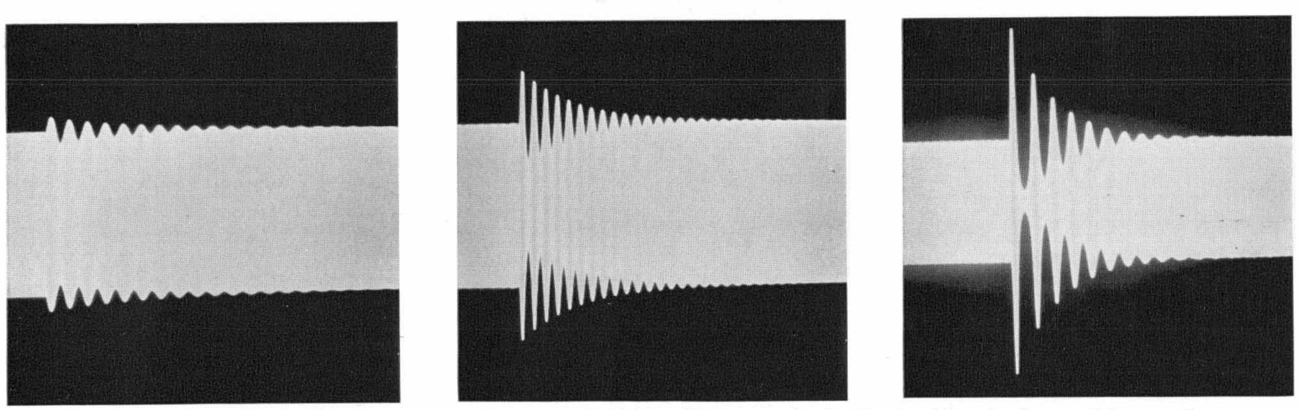
For sinusoidal outputs, the square law term requires only additional d-c and second harmonic input components and hence does not affect the fundamental gain, μ. The first term of interest to us after the linear term is therefore the cubic term. Further, in the case of small non-linearity all higher terms are generally negligible by comparison. Accordingly we will as-



(a) Envelope response typical of RC oscillator to slight amplitude disturbances within circuit. Oscillation frequency is 100 cps (left), 1 kc (middle), and 10 kc (right). Sweep times are 200, 100, and 50 millisec/cm, respectively. Oscillator distortion is 66 db below oscillation level.



(b) Envelope response to internal disturbance when lamp circuit time constant is increased, showing slower envelope transients. Distortion, sweep times, and oscillator frequencies are same as in (a).



(c) Envelope response when oscillation level is reduced to obtain very high effective linearity in amplifier portion of circuit. Distortion is about 90 db below oscillation level. Sweep times are 1 sec/cm (left), 500 millisec/cm (middle), and 100 millisec/cm (right). Oscillator frequencies are same as in (a).

Fig. 7. *Oscillograms of RC oscillator envelope responses made to test theory developed in accompanying article.*

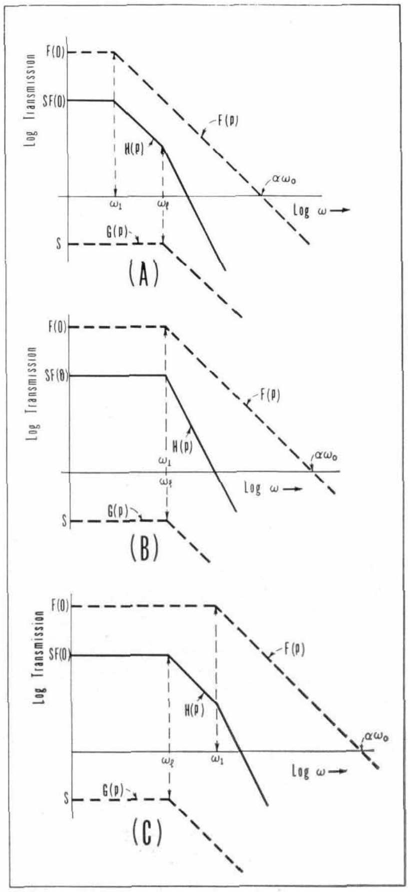


Fig. 6. *Envelope loop gain characteristic H(p) for conditions where pole in envelope response resulting from amplifier non-linearity is (a) below, (b) equal to, and (c) above frequency corresponding to lamp circuit time constant. (a) and (c) represent stable conditions since gain slope is only 6 db/octave over effective region. Most oscillatory response case (b) actually occurs in practice only at very low oscillator frequencies of a few cycles and at envelope frequencies on the order of 1 cps.*

sume as the transfer characteristic:

$$e_{in} = \frac{1}{\mu_0}\left(e_{out} + a e^3_{out}\right)$$

where μ_0 is the amplifier gain for infinitesimal inputs.

Now assume $e_{out} = E \sin \omega_0 t$.

Then

$$e_{in} = \frac{1}{\mu_0}\left[E \sin \omega_0 t + a E^3 \sin^3 \omega_0 t\right]$$

$$= \frac{1}{\mu_0}\left[E\left(1 + \frac{3aE^2}{4}\right)\sin \omega_0 t - \frac{aE^3}{4}\sin 3\omega_0 t\right]$$

If D = 3rd harmonic distortion without feedback, then

$$D = \frac{\frac{aE^2}{4}}{1 + \frac{3aE^2}{4}}, \qquad (24a)$$

$$\mu = \frac{\mu_0}{1 + \frac{3aE^2}{4}}, \qquad (24b)$$

$$\frac{D\mu_0}{\mu} = \frac{aE^2}{4}. \qquad (24c)$$

Suppose now that such an amplifier is connected as shown in Fig. 4, and is initially oscillating with $E = E_1, R_1' = \overline{R_1}$, and $\mu = \mu_1$. If R_1' is now changed to $\overline{R_1} + \delta R_1'$, k will change by an amount

$$\Delta k = \left.\frac{\partial k}{\partial R_1}\right|_{R_1 = \overline{R_1}} \delta R_1' = k_1(1 - k_1)\epsilon$$

where, again, $\epsilon = \frac{\delta R_1'}{\overline{R_1}}$, and k_1 is the value of k_0 as given by equation (8) with $\mu = \mu_1$. The oscillation will now stabilize at a new amplitude, such that the change in k_0 required by (8) has the above value.

$$k_2 - k_1 = \Delta k$$

$$\frac{1}{\mu_1} - \frac{1}{\mu_2} = k_1(1 - k_1)\epsilon$$

Using (24b) we find

$$\left[1 + \frac{3aE_1^2}{4}\right] - \left[1 + \frac{3aE_2^2}{4}\right] = \mu_0 k_1(1 - k_1)\epsilon$$

and, letting $\Delta(E^2) \equiv E_2^2 - E_1^2$, this reduces to

$$\frac{\Delta(E^2)}{E_1^2} = -\frac{4}{3aE_1^2}\mu_0 k_1(1 - k_1)\epsilon$$

$$= -\frac{\mu_1}{3D_1}k_1(1 - k_1)\epsilon$$

where D_1 is the distortion, D, at $E = E_1$. Dividing both sides by ϵ, we find the d-c envelope (squared) transmission to be

$$F(0) = -\frac{\mu_1}{3D_1}k_1(1 - k_1)$$

The distortion with feedback, d, is more generally known, and it is appropriate to convert to this quantity. Since at the third harmonic, $z = \pm j3$, we have

$$d_1 = \frac{D_1}{\left|1 - \mu_1\beta\right|_{z = \pm j3}},$$

and from (4) and (8) we find

$$D_1 = \frac{n}{n + 2}\,\frac{d_1\mu_1}{\gamma}$$

where

$$\gamma = \sqrt{1 + \left(\frac{3(n + 2)}{8}\right)^2}$$

and is the network loss at $z = \pm j3$ relative to the loss at $z = 0$ or $z = \infty$. Therefore

$$F(0) = -\frac{\gamma k_1(1 - k_1)}{3d_1}\,\frac{n + 2}{n} \qquad (25)$$

Now the initial *rate* of change of E^2 (at $E = E_1$) is not affected by the non-linearity since the amplitude must first change for the non-linearity to be felt; so we may differentiate (19) to obtain

$$\frac{1}{E_1^2}\left.\frac{d(E^2)}{dt}\right|_{t=0} = -\alpha_1\omega_0\epsilon$$

where α_1 is the value of α with $\mu = \mu_1$. Further, the rate of change of E^2 will be proportional to $E^2 - E_2^2$ since the excess input to the amplifier (caused by excess k) is proportional to this quantity. As a result, the transient in E^2 will be exponential. The time function which fits all these conditions is

$$\frac{E^2}{E_1^2} = 1 + F(0)\left[1 - \exp\left(\frac{\alpha_1\omega_0 t}{F(0)}\right)\right]\epsilon.$$

The response to a *unit* step is therefore

$$\frac{\Delta(E^2)}{E_1^2} = F(0)\left[1 - \exp\left(\frac{\alpha_1\omega_0 t}{F(0)}\right)\right]$$

and corresponds to the frequency function

$$F(p) = -\frac{\alpha_1\omega_0}{p - \frac{\alpha_1\omega_0}{F(0)}} = -\frac{\alpha_1\omega_0}{p + \omega_1} \qquad (26)$$

Since

$$\frac{\alpha_1}{k_1(1 - k_1)} = \left(\frac{\partial b}{\partial k}\right)_{k = k_1} = \frac{(n + 2)^2}{n},$$

we find using (25) that

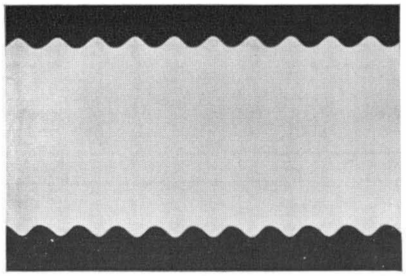
$$\omega_1 = \frac{3(n + 2)}{\gamma}d_1\omega_0 \qquad (27)$$

As a result of the non-linearity, the envelope response has a pole at $p = -\omega_1$ rather than at the origin (see eq. (20)). The total envelope loop gain $H(p)$ now consists of two simple RC cutoffs, one fixed at ω_ℓ, the other at ω_1 and therefore variable with ω_0. Figs. 6a, 6b, and 6c show the factors $F(p)$ and $G(p)$ and their product $H(p)$ for three oscillator frequencies which place ω_1 below, equal to, and above ω_ℓ.

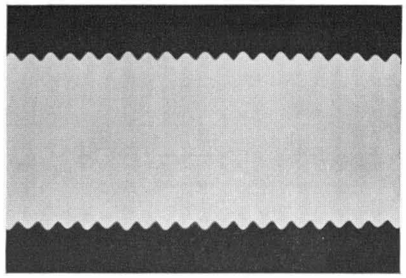
From the figures it is clear that for $\omega_1 << \omega_\ell$ or $\omega_1 >> \omega_\ell$, the loop gain has a simple 6db/octave cutoff over the region of interest. The least phase margin, and therefore the most oscillatory condition, occurs when $\omega_1 = \omega_\ell$. We then have

$$F(p) = F(0)\frac{\omega_\ell}{p + \omega_\ell}$$

$$G(p) = S\frac{\omega_\ell}{p + \omega_\ell}$$



(a)
Sixty-cycle hum modulation peak occurring at oscillator frequency of about 60 kc.



(b)
One hundred and twenty cycle hum modulation peak occurring at oscillator frequency of 240 kc.

Fig. 8. *Oscillograms of maximum enhancement of hum in RC oscillator envelope when amplifier circuit has very high linearity discussed in article.*

and the closed loop response to disturbance in R_1 is:

$$T(p) = \frac{F(p)}{1 - F(p)G(p)}$$

$$= F(0)\,\frac{\omega_\ell (p + \omega_\ell)}{p^2 + 2\omega_\ell p - S\,F(0)\,\omega_\ell^2} \qquad (28)$$

If the distortion is small,

$$\left| S\,F(0) \right| >> 1 \, ,$$

and the maximum occurs near

$$p = \pm j\sqrt{-\,S\,F(0)}\;\omega_\ell$$

(Note: $F(0)$ itself is negative.) The value of this maximum is

$$\left|T\right|_{max} \approx \left|\frac{F(0)}{2}\right|$$

$$\left.\begin{aligned} &\approx \frac{\gamma k_1 (1 - k_1)}{3 d_1}\;\frac{n + 2}{2n} \\ &\approx \frac{\alpha_1 \gamma}{6 d_1 (n + 2)} \end{aligned}\right\} \qquad (29)$$

and it occurs at a modulation frequency of

$$\omega_m = \sqrt{\frac{S\,\alpha_1 \gamma}{3 d_1 (n + 2)}} \qquad (30)$$

For the typical case of $\mu_1 >> 1$, $S = 1/4$, and $n = 2$, we find

$$\gamma = \sqrt{1 + \left(\frac{3}{2}\right)^2} = \frac{\sqrt{13}}{2}$$

$$\alpha_1 = 2$$

$$\left|T\right|_{max} = \frac{\sqrt{13}}{24 d_1} = \frac{1}{6.66 d_1}$$

$$\omega_m = \frac{\omega_\ell}{\sqrt{13.32 d_1}}$$

If the distortion is assumed to be 50 db down, then $d_1 \approx 10^{-\frac{5}{2}}$ and

$$\left|T\right|_{max} = 47.5 \approx 33.5 \text{ db} \, .$$

Comparing this result with the linear case, we see that third harmonic distortion 50 db down has reduced the envelope gain enhancement by about 100 db! Further, the worst situation, in the example above, now occurs at the bottom of the tuning range at

$$\alpha_1 \omega_0 \approx \left|F(0)\right| \omega_\ell \, ,$$

or

$$f_0 = 47.5 \times 0.2 = 9.5 \sim$$

and the modulation frequency which is enhanced is

$$f_m = 0.975 \sim \, ,$$

which is well below any hum or strong microphonic component.

SENSITIVITY TO μ VARIATIONS

The closed loop envelope transmission functions $T_0(p)$ and $T(p)$ which we have used in evaluating the relative envelope loop stability in the linear and non-linear cases are correct only when the disturbance is a perturbation in R_1. Such perturbations might be caused by filament vibration or deliberate adjustment of a variable resistor in series with the lamp. The response, $T_q(p)$, to fractional perturbations of some other parameter, q, will be of the form

$$T_q(p) = \frac{q\,\dfrac{\partial k}{\partial q}}{\left(R_1\,\dfrac{\partial k}{\partial R_1}\right)}\,T(p) \qquad (31)$$

For example, if n changes (during tuning) we have

$$T_n(p) \approx \frac{\dfrac{2n}{(n + 2)^2}}{k(1 - k)}\,T(p)$$

or if the disturbances are variations in μ we find

$$T_\mu(p) = \frac{1}{\mu k (1 - k)}\,T(p) \qquad (32)$$

Thus

$$\frac{\Delta(E^2)}{E^2} = \frac{1}{\mu k (1 - k)}\,T(p)\,\frac{\delta\mu}{\mu}$$

which apparently decreases as μ is increased. However the larger μ is made, the less in general will be the distortion, d, and the greater will be $|T|_{max}$ as given by equation (29). Thus the worst enhancement, $|T\mu|_{max}$, is independent of μ to a first order so long as $\mu >> 1$, and we are led to the important conclusion that the distortion may be reduced indefinitely by increasing μ without worsening the enhancement of μ variations at any frequency.

EXPERIMENTAL RESULTS

To test the above theory, a transient was produced in an -*hp*- Model 200 CD 5 cps - 600 kc oscillator. The resulting envelope transient was then observed at various oscillator frequencies and at two widely different output levels corresponding to different degrees of amplifier linearity. The transient was produced by shorting a small resistor, δR_1, in series with the lamp (or lamps). In all cases δR_1 was made ½% of R_1, so the input disturbance was a step function of magnitude

$$\delta R_1 / R_1 = 0.005.$$

The envelope responses to this transient are shown in the accompanying sets of oscillograms. The first set, Figs. 7(a), shows the envelope transient at normal oscillator operating level and at oscillator frequencies of 100 cps, 1 kc, and 10 kc. The normal complement of two lamps was used for R_1. The measured distortion was about 66 db below the fundamental and consisted almost entirely of 3rd harmonic. The results show an oscillatory transient at the lower frequencies and a non-oscillatory response at 10 kc, indicating an approach to a 0 - 6 db/octave loop gain characteristic as ω_0 is increased, in accord with the non-linear theory.

The oscillator was now modified by adding two more lamps to R_1 to make a total of four in series. R_2 was then readjusted to restore the output to the normal level. The measured distortion was the same as before, and as can be seen from Figs. 7(b), the transients are quite similar, but slower, owing to the longer time constant of the lamps at reduced temperature.

Next, all the lamps but one were removed and R_2 reduced by a factor of four. This produced one-fourth the normal oscillation level and hence the same lamp temperature as in the immediately previous case. Thus the only significant change was the distortion, which dropped to a level too low to measure accurately but which was about 90 db below the fundamental. From Fig. 7(c), it is obvious that the envelope transient is now more oscillatory in all cases, but especially so for the higher oscillator frequencies as one would expect from the linear theory. (See also Fig. 2, front page.)

A search was now made for the large enhancement of hum modulation to be expected from the linear theory. At an oscillator frequency of 60 kc the 60-cycle modulation shown in Fig. 8(a) was found. At four times this frequency or 240 kc the 120-cycle modulation shown in Fig. 8(b) was also found. Neither of these modulations is present at the normal operating level (and distortion). Their appearance at very low distortion levels nicely illustrates the drastic decrease in envelope stability at high oscillator frequencies as one approaches perfect μ-circuit linearity.

CONCLUSION

Simple lamp stabilized oscillators depend in large measure for their dynamic amplitude stability upon slight non-linearity (compression) in the associated amplifier. The effectiveness of this non-linearity in stabilizing the envelope transient response is so great that instability is not serious unless the distortion is more than 70 or 80 db down. For oscillators having less distortion than this, more sophisticated AVC methods are probably desirable.

—Bernard M. Oliver

UTILIZING VLF STANDARD BROADCASTS WITH THE *-hp-* FREQUENCY DIVIDER AND CLOCK

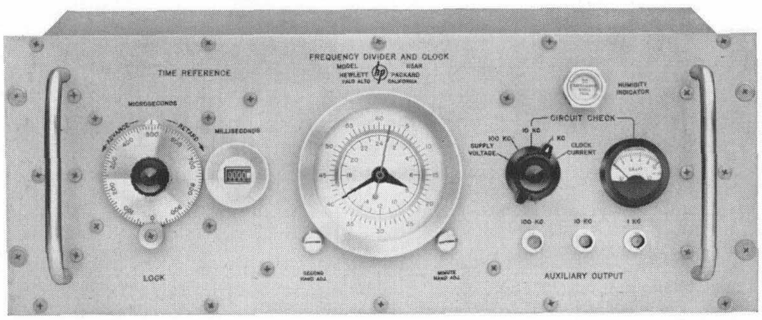


Fig. 1. -hp- *Model 113AR Frequency Divider and Clock increases the precision with which local frequencies can be compared with broadcast standard.*

Recently, it was shown[1] here how both frequency and time can be locally determined to increased precision by employing a specially-developed -hp- Frequency Divider and Clock. The method discussed utilized the standard time signals which are broadcast in the HF region.

The discussion also mentioned that the Clock was similarly valuable for enabling the time-comparison technique to be used with time signals broadcast at VLF, such as from station NBA[2]. Several stations are now broadcasting standard signals at VLF, and much interest has been shown in using the Clock for comparing local frequencies with VLF signals. Being ground-wave propagated, VLF signals offer the advantage for this work that they are virtually free of the transmission-path effects that make it necessary with HF signals for comparisons to be separated by a period of hours or days to achieve high comparison accuracy. With VLF signals it is possible in an hour or so to make a comparison to a few parts in 10^{10}.

Determinations of time with VLF signals can be made as described previously by comparing the pulse from the Clock with the 1 pps modulation on the VLF transmissions. The increased-precision frequency comparisons, however, are made by comparing the pulse from the Clock with the phase of the VLF carrier frequency. An equipment arrangement for doing this is indicated in Fig. 2. In this set-up the pulse from the Clock is positioned by the phase shifter on the Clock panel to occur in the middle or late portion of the received VLF pulse. The Clock pulse is then used to start a time-interval measurement on the counter. The measurement will be stopped by the next following cycle of the carrier frequency from the received VLF signal. The resulting time-interval readings are printed by the Model 560A Digital Recorder and continuously-plotted on a strip-chart recorder using the analog output from the 560A. A chart is thus obtained which shows the relative time drift between the clock pulse and the VLF carrier cycle. This chart will have a resolution of 2 or 3 microseconds[3], corresponding to a frequency comparison of a few parts in 10^{10} in one hour.

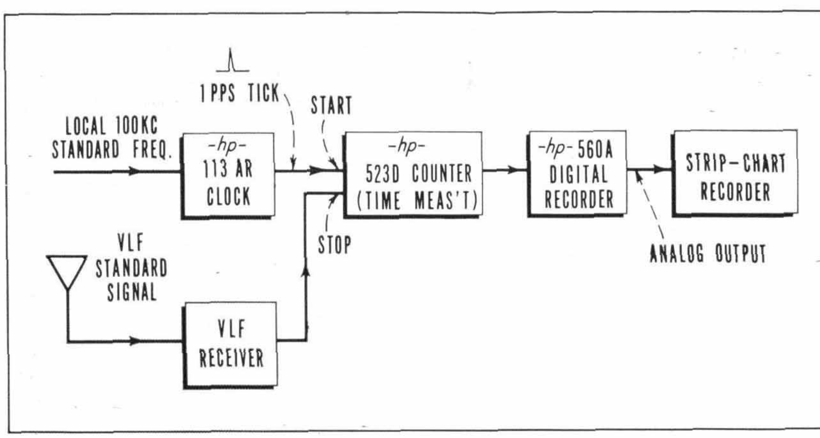


Fig. 2. *Equipment arrangement for comparing local standard frequency with VLF standard broadcasts.*

[1]Dexter Hartke, "A New Clock for Improving the Accuracy of Local Frequency And Time Standards," Hewlett-Packard Journal, Vol. 11, No. 3-4, Nov.-Dec., 1959.

[2]Station NBA, Panama, broadcasts on an 18 kc carrier frequency which is modulated at a 1 pps repetition rate. Both the carrier frequency and modulation repetition rate are monitored by the Naval Research Laboratory and held to an accuracy of 2 parts in 10^{10} with respect to a cesium standard. Vlf experimental broadcasts are also being made at reduced power by the National Bureau of Standards on 20 and 60 kc with a later increase to full power expected.

[3]Final resolution depends on such factors as receiver location, type of receiver, etc. At the -hp- laboratories a Model 302A Wave Analyzer has been successfully used as a receiver for both NBA, Panama, and WWVL, Boulder, Colo., with a simple five-foot diameter loop for an antenna.